Title: Introduction to Hochschild (co)homology.

Abstract: First we will introduce the notions of Hochschild homology and cohomology. Their different applications in representation theory, topology and geometry will be discussed. More precisely,we will review several conjectures (Happel's conjecture, Han's conjecture, strong no loop conjecture) in representation theory. We will also talk about its relationship with deformation quantization.

Talk1: We first give the definitions of Hochschild homology and cohomology. Then we will give several motivations from different points of view on why we study Hochschild homology and cohomology. For instance, it is related to Auslander-Reiten conjecture, Deligne conecture, Han's conjecture and deformation quantization. It plays a crucial role in non-commutative geometry.

Talk2: We discuss operations on Hochschild chain and cochain complexes. More precisely, we will prove that the Hochschild cohomology ring is a Gerstenhaber algebra. We will explain that these operations are closely related to deformation theory.

Talk3: We focus on commutative algebras during this talk. For a commutative algebra, we introduce the Hochschild-Kostant-Rosenberg (HKR) maps from Hochschild chain complex to differential forms, and dually from poly-vector fields to Hochschild cochain complex. We will prove that they are quasi-isomorphisms for smooth algebras.

Talk4: We introduce the notion of deformation quantization. We explain how Kontsevich proved the deformation quantization theorem by proving that the HKR map of Hochschild cochain complex can be extended to an L-infinity quasi-isomorphism. We will also give an application of deformation quantization to Duflo isomorphism from Lie theory.

Talk5: We introduce the notion of Tate-Hochschild cohomology, which is analogous to Hochschild cohomology. We will show several recent developments on this cohomology.

Title: Lie theory associated to weighted projective lines

Abstract: The notion of weighted projective lines was invented by Geigle and Lenzing, motivated by giving a geometric treatment for Ringel's canonical algebras. By a deep theorem of Happel, up to derived equivalence, a hereditary category with a tilting object is either equivalent to the module category of a path algebra, or equivalent to the category of coherent sheaves on a weighted projective line.

The study of weighted projective lines is related to many mathematical areas, for example, the representation theory of algebras, noncommutative algebraic geometry, Lie theory, and singularity theory.

In this talk we mainly study the Lie theory associated to weighted projective lines via Ringel–Hall algebra approach. The Ringel–Hall algebra of the category of coherent sheaves and its Drinfeld double have been studied by many people, for example, Burban, Kapranov, Schiffmann, Xiao etc. According to their work, the double composition algebra (certain subalgebra of the Ringel–Hall algebra) provides a realization of the quantized enveloping algebra of the loop algebra of a Kac–Moody algebra. By using the Ringel–Hall Lie algebra construction introduced by Peng–Xiao, Crawley-Boevey gave a realization of the loop algebras of Kac–Moody algebras, and obtained a Kac-type theorem for weighted projective lines, which describes the dimension types of indecomposable coherent sheaves in terms of the root systems for the loop algebras.

Content:

(1) The category of coherent sheaves on projective line;
(2) Introduction on weighted projective lines;
(3) The category of coherent sheaves on weighted projective lines;
(4) Ringel–Hall algebra approach to quantized loop algebras;
(5) Kac's theorem for weighted projective lines.

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Cluster algebras

September 13, 2017

This series of talks gives a foundamental introduction to cluster algebras, which is a relatively young field in mathematics. We present basic definitions and collect important results in this area, and we select and introduce several fruitful approaches to cluster algebras.

1 Basics of cluster algebras

In this talk, we introduce the definitons and background of cluster algebras. And we compute examples in details.

2 Cluster algebras and categorification

We introduce the theory of cluster categories. We discuss how to use these categories to study cluster algebras.

3 Cluster algebras and scattering diagrams

We present basics of scattering diagrams arising from geometry. We explain how cluster algebras are related to these diagrams, following the work of Gross–Hacking–Keel–Kontsevich.

4 Cluster algebra and τ-tilting theory

We introduce the τ-tilting theory. We shall see how the notion of mutation is generalized to τ-tilting modules of finite dimensional algebras.